

## Review for the Final Exam

### A. Regular Languages

- DFAs, NFAs,  $\epsilon$ -NFAs You should be able to convert any of the others to a DFA.
- Regular Expressions. It is fairly easy to convert a regular expression to a DFA. It is possible but harder to convert a DFA to a regular expression.
- The Pumping Lemma: If  $|w| > p$  then  $w=xyz$  where  $|xy| \leq p$ ,  $y$  is not empty, and  $xy^iz$  is in the language for all  $i \geq 0$ .
- Properties of Regular Languages: Unions, Intersections, Differences and Complements of regular languages are regular.

### B. Context-Free Languages

- Grammars
- PDAs
- To show that grammars generate the same languages as PDAs we found algorithms to convert a grammar to a PDA (easy) and to convert a PDA to a grammar (hard).
- Chomsky Normal Form and the algorithm for finding a CNF grammar equivalent to a given grammar.
- The Pumping Lemma for Context-Free languages: If  $|z| > p$  then  $z=uvwxy$  where  $|vwx| \leq p$ ,  $v$  and  $x$  aren't both empty, and  $uv^iwx^iy$  is in the language for all  $i \geq 0$ .
- Properties of CF Languages: Unions and concatenations of CF languages are CF. Intersections and Complements of CF languages are not necessarily CF.

### C. Turing Machines

- Simple TMs, multi-track, multi-tape and non-deterministic TMs
- Church's Thesis: TMs embody our notion of an algorithm

### D. Decidability

- Recursive languages, Recursively enumerable languages, Decidable problems, Recognizable problems
- The diagonal language  $\mathcal{L}_d = \{ \langle M \rangle \mid M \text{ does not accept its own encoding} \}$  is not RE but its complement is RE.
- The universal language  $\mathcal{L}_u = \{ \langle M, w \rangle \mid M \text{ accepts } w \}$  is RE but not Recursive. The complement of  $\mathcal{L}_u$  is not RE.
- The halting language  $\mathcal{L}_{\text{halt}} = \{ \langle M, w \rangle \mid M \text{ halts on input } w \}$  is RE but not recursive.
- Rice's Theorem: Any nontrivial property of context-free languages is undecidable.
- Reductions: To show that a language  $\mathcal{L}$  is not RE or is not Recursive, reduce a language  $\mathcal{L}^*$  that you know is not RE or is not Recursive to it. This means showing that if you have a recognizer or decider for  $\mathcal{L}$ , then you would also have one for  $\mathcal{L}^*$

#### E. NP-Completeness

- $\mathcal{P}$  is the class of problems that can be solved deterministically in polynomial time
- $\mathcal{NP}$  is the class of problems that can be solved non-deterministically in polynomial time, which usually means that a solution can be verified deterministically in polynomial time.
- Cook's (or Cook-Levin) Theorem: SAT is NP-Complete.
- You should know what all of this means, but I am unlikely to ask you to prove that a specific language is NP-Complete.